

Answer all questions. Calculators and Mobile Phones are not allowed.

1. (3 pts. each) Evaluate the following integrals:

(a) $\int \frac{4x^2 - 3x + 2}{4x^2 - 4x + 3} dx.$

(b) $\int \frac{1}{x+5-5\sqrt{x+1}} dx.$

(c) $\int x\sqrt{8-2x-x^2} dx.$

(d) $\int \cot^3 x \csc^{-\frac{3}{2}} x dx.$

(e) $\int x^2(\ln x)^2 dx.$

2. (3 pts.) Check the improper integral for convergence, and find its limit if it is convergent.

$$\int_{-\infty}^{\infty} \frac{1}{e^x + e^{-x}} dx.$$

3. (3 pts.) By using implicit differentiation, find $\frac{dy}{dx}$, if

$$\tanh(xy) + y^2(1 + \cosh^2 x)^x + \ln(1 + \sinh x) = e$$

4. (2 pts. each) Find the limit, if exists.

(a) $\lim_{x \rightarrow \infty} (1 - e^{-2x})^{\ln x}.$

(b) $\lim_{x \rightarrow 1} \left(\frac{1}{x-1} - \frac{1}{\ln x} \right).$